NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2024

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

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SECTION A

QUESTION 1

(a)	$AO = OE$ (radii) $\hat{E}_1 = 40°$ (∠opp = radii) $\hat{D} = 140°$ (opp∠cyclic quad) $\hat{F}_1 = 90°$ (line from cent bisects chord) $\hat{A}_1 = 50°$ (tan ⊥ rad) $\hat{B} = 50°$ (tan chord th) $A\hat{O}E = 100°$ (∠ @ cent) / (int ∠ Δ)	reason reason reason reason reason reason
(b)	Construction: radii OA and OC $O\hat{A}C = 90° - \hat{A}_1$ (tan ⊥ rad) $O\hat{C}A = 90° - \hat{A}_1$ (∠ opp = radii) $A\hat{O}C = 2\hat{A}_1$ (int ∠ Δ) $\hat{B} = \hat{A}_1$ (∠@cent)	

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QUESTION 2

(a)	$\left(3;-\frac{9}{2}\right)$	*y* *x*
(b)	$AM = \sqrt{(-3-3)^2 + (-2-2)^2}$ $AM = 2\sqrt{13}$	Sub correct formula answer
(c)	$m_{MA} = \frac{-2-2}{-3-3}$ $\tan\theta = \frac{2}{3}$ $\theta = 33{,}69°$	m_{MA} $\tan\theta = \frac{2}{3}$ $\theta = 33{,}69°$
(d)	$y - \frac{9}{2} = \frac{2}{3}(x+3)$ $\therefore y = \frac{2}{3}x + \frac{13}{2}$	sub pt gradient answers in correct form
(e)	$PM = 6{,}5\,units$ $MR = 6{,}5\,units$ $\therefore PM = MR$ $\therefore$ PARM is a rhombus (adj sides =) **Alternate:** $m_{PR} = \frac{\frac{9}{2}+\frac{9}{2}}{-3-3}$ $m_{PR} = -\frac{3}{2}$ $\therefore m_{MA} \times m_{PR} = -1$ $\therefore\therefore$ PARM is a rhombus (diagbisect @ 90°)	PM MR Correct method used to prove a rhombus

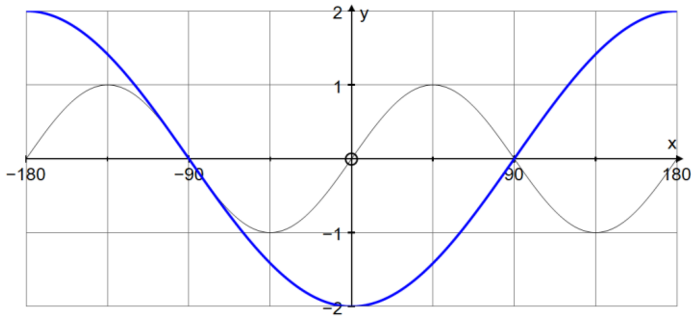
QUESTION 3

(a)	$x^2 - 2x + 1 + y^2 + 2y + 1 = 3 + 1 + 1$ $(x-1)^2 + (y+1)^2 = 5$ $\therefore N(1;-1)$ and $r = \sqrt{5}$ units	Completing the square N r
(b)	$x^2 - 2x + y^2 + 2y - 3 = 0$ *sub* $y = 0$ $x^2 - 2x + (0)^2 + 2(0) - 3 = 0$ $x^2 - 2x - 3 = 0$ $x = 3$ or $x = -1$ $\therefore K(-1;0)$	quadratic equ solutions for x $K(-1;0)$
(c)	$m_{KN} = \frac{-1-0}{1+1}$ $m_{KN} = -\frac{1}{2}$ $\therefore m_{KJ} = 2$ $y = 2(x+1)$ $\therefore y = 2x + 2$	m_{KN} m_{KJ} $y = 2x + 2$
(d)	$\tan\theta = 2$ $\theta = 63{,}43°$ $\therefore \hat{L} = 63{,}43°$ (tan chord th)	$\tan\theta = 2$ θ $\hat{L}$ and reason

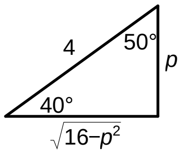
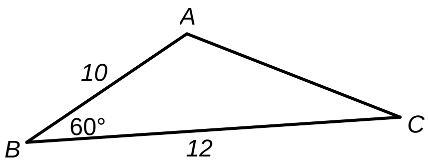
QUESTION 4

(a)	$\hat{M} = 90^\circ$ ($\angle$in half circle)	M reason
(b)	In ΔLMN and ΔLPM (1) $\hat{L} = \hat{L}$ (common) (2) $\hat{M} = \hat{P}_1$ (both = 90°) (3) $\hat{M}_1 = \hat{N}$ (3rd$\angle$) $\therefore \Delta LMN \mid\mid\mid \Delta LPM$ (*AAA*)	$\hat{L} = \hat{L}$ (common) $\hat{M} = \hat{P}_1$ *AAA*
(c)	$\frac{LM}{LP} = \frac{MN}{PM} = \frac{LN}{LM}$ $LM^2 = LP \times LN$ $4^2 = LP \times 8$ LP = 2 **Alternate:** $4^2 = LP \times 8$ ($\perp$from right $\angle$'d vertex) LP = 2	$\frac{LM}{LP} = \frac{MN}{PM} = \frac{LN}{LM}$ $4^2 = LP \times 8$ LP = 2 $4^2 = LP \times 8$ $\perp$from right $\angle$'d vertex LP

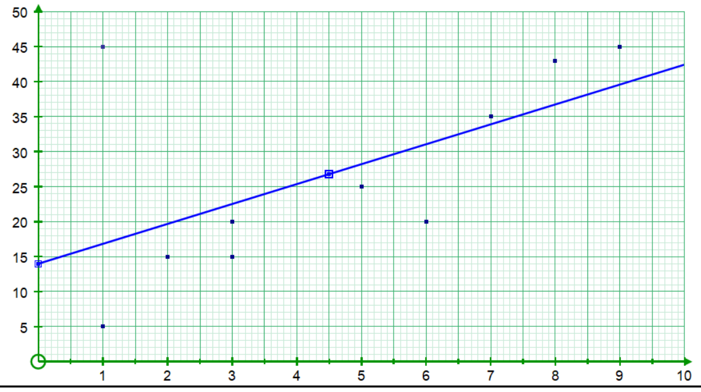
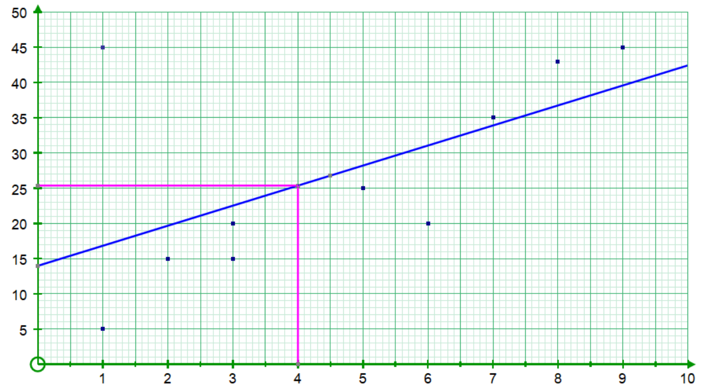
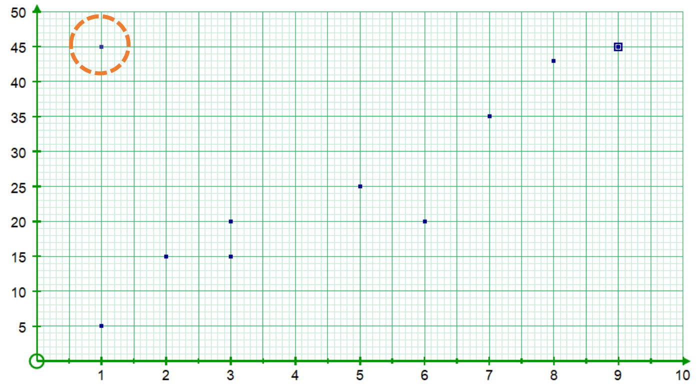
QUESTION 5

(a)	$b = 2$	b
(b)	$180°$	answer
(c)		cos shape consistent amp correct x and y cuts
(d)	2	answer
(e)	$\theta = 90° + k180°$	$90°$ $k180°$
(f)(1)	$x \in [-90°;90°]$	$-90°;90°$
(f)(2)	$x \in (0°;90°) \cup (90°;180°)$	$(0°;90°)$ $(90°;180°)$
(f)(3)	$\therefore x \in (0°;180°)$	interval brackets
(g)	$y = -2\cos(x - 45°) + 2$	$-45°$ $+2$

QUESTION 6

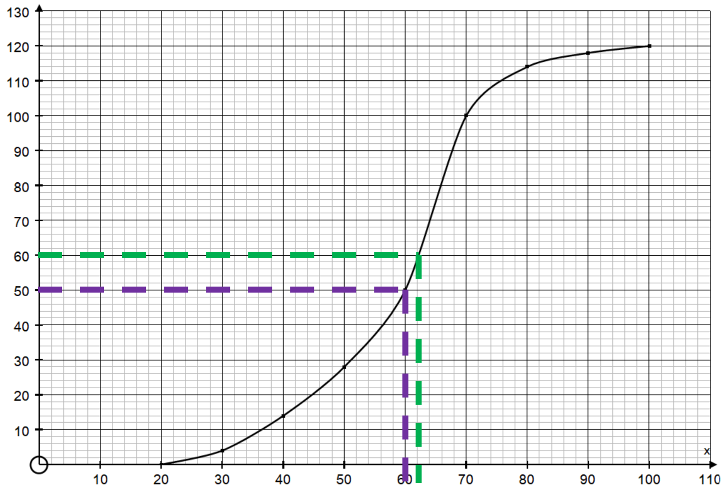
(a)(1)	$= \sin(50° - 10°)$ $= \sin 40°$ $= \dfrac{p}{4}$	$(50° - 10°)$ / $\sin 40°$ answer
(a)(2)	$= \cos 2(20°)$ $= \cos 40°$ $= \dfrac{\sqrt{16 - p^2}}{4}$  	Adj on diagram $\cos 40°$ answer
(a)(3)	$= \dfrac{\sqrt{16 - p^2}}{p}$	answer
(b)	 $AC^2 = 10^2 + 12^2 - 2(10)(12)\cos 60°$ $AC = 2\sqrt{31}$ units	Diagram Cos rule answer

QUESTION 7

(a)	$y = 13{,}98 + 2{,}85x$	A B
(b)	$(\bar{x};\bar{y}) = (4{,}5\,;\,26{,}8)$ 	$\bar{x}$ $\bar{y}$ $y-\text{int}$ line through $(\bar{x};\bar{y})$ and $y-\text{int}$
(c)	R25 000 	Indicate on graph answer in thousands of rands
(d)	$r = 0{,}58$	answer
(e)		Circled on diagram
(f)	r will increase	answer

SECTION B

QUESTION 8

<table>
<tr>
<td>(a)</td>
<td>

Original Q_2 – new Q_2 = 62 – 60 = 2</td>
<td></td>
<td>Median positions at 60 and 50
62
60</td>
</tr>
<tr>
<td>(b)</td>
<td>data is skewed left
$\therefore \bar{x} < Q_2$</td>
<td>✓
✓</td>
<td>left
$\bar{x} < Q_2$</td>
</tr>
<tr>
<td>(c)</td>
<td>$4 - \bar{x} = -3$
$\therefore \bar{x} = 7$

$\dfrac{4 + 8 + p}{3} = 7$
$\therefore p = 9$

<table>
<tr><th>x</th><th>$x - \bar{x}$</th><th>$(x - \bar{x})^2$</th></tr>
<tr><td>4</td><td>–3</td><td>9</td></tr>
<tr><td>8</td><td>1</td><td>1</td></tr>
<tr><td>p</td><td>2</td><td>4</td></tr>
</table>

$\sigma = \sqrt{\dfrac{9 + 1 + 4}{3}}$
$\therefore \sigma = 2{,}16$</td>
<td>✓
✓
✓
✓</td>
<td>$\bar{x} = 7$
$p = 9$
Completing the table
$\sigma = 2{,}16$</td>
</tr>
</table>

QUESTION 9

(a)	$D\hat{F}E = 90^\circ \left(\angle \text{ in } \frac{1}{2} \text{ circle}\right)$ $\therefore D\hat{F}E = D\hat{E}G \text{ (both} = 90^\circ)$ $\therefore EG \parallel FC \text{ (converse corres } \angle\text{'s)}$	$D\hat{F}E = 90^\circ$ $\angle \text{ in } \frac{1}{2} \text{ circle}$ $D\hat{F}E = D\hat{E}G$
(b)	In ΔABC and ΔDFC (1) $\hat{C}_1 = \hat{D}$ (tan chord th) (2) $\hat{C}_2 = \hat{A}$ (tan chord th) (3) $AC = CD$ (given) $\therefore \Delta ABC \equiv \Delta CFD \, (AAS)$ $\therefore AB = CF = 10$ units **Alternate**: In ΔABC and ΔDFC (1) $\hat{B} = 90^\circ \left(\angle \text{ in } \frac{1}{2} \text{ circle}\right)$ $\therefore \hat{B} = \hat{F}$ (both $= 90^\circ$) (2) $\hat{C}_2 = \hat{A}$ (tan chord th) (3) $AC = CD$ (given) $\therefore \Delta ABC \equiv \Delta CFD \, (AAS)$ $\therefore AB = CF = 10$ units	$\hat{C}_1 = \hat{D}$ tan chord th $\hat{C}_2 = \hat{A}$ tan chord th $AC = CD$ *AAS*
(b)	$3(10) = 5DE$ $DE = 6$ $FD^2 = 26^2 - 10^2$ (pythag) $FD = 24$ $\therefore FE = 24 - 6 = 18$	$DE = 6$ $FD = 24$
(d)	$\frac{CG}{26} = \frac{18}{24}$ (prop th EG‖FC) $\therefore CG = 19{,}5$ units	$\frac{CG}{26} = \frac{18}{24}$ prop th EG‖FC $CG = 19{,}5$ units

QUESTION 10

(a)	Sides in proportion	reason
(b)	$\hat{D}_1 = \hat{A}(\vert\vert\vert)$ $\therefore$ DF is a tangent to the circle at D (converse tan chord th)	$\hat{D}_1 = \hat{A}$ converse tan chord th
(c)	$\hat{C} = \hat{F}(\vert\vert\vert)$ $\therefore$ GCFD is a cyclic quad(converse $\angle$'s same seg)	$\hat{C} = \hat{F}$ converse $\angle$'s same seg

QUESTION 11

(a)	$C\hat{A}B = 90° - \theta(\text{int}\angle\Delta)$ $\frac{AC}{\sin(180° - \theta)} = \frac{10}{\sin(90° - \theta)}$ $AC = \frac{10\sin\theta}{\cos\theta}$ $AC = 10\tan\theta$	$90° - \theta$ $\frac{AC}{\sin(180° - \theta)}$ $\frac{10}{\sin(90° - \theta)}$ $\frac{10\sin\theta}{\cos\theta}$
(b)	$\tan\theta = \frac{DC}{AC}$ $\tan\theta = \frac{15}{10\tan\theta}$ $\tan^2\theta = \frac{3}{2}$ $\tan\theta = \pm 1{,}22 \ldots$ $\theta = 50{,}77°$	tan ratio sub of AC $\tan^2\theta = \frac{3}{2}$ $\theta = 50{,}77°$

QUESTION 12

(a)	LHS: $= \dfrac{\cos 2x + \cos x}{\sin 2x - \sin x}$ $= \dfrac{2\cos^2 x + \cos x - 1}{2\sin x \cos x - \sin x}$ $= \dfrac{(2\cos x - 1)(\cos x + 1)}{\sin x(2\cos x - 1)}$ $= \dfrac{\cos x + 1}{\sin x}$ $\therefore$ LHS = RHS	$\cos 2x$ $+\cos x$ $-\sin x$ $2\cos^2 x - 1$ $2\sin x \cos x$ $(2\cos x - 1)(\cos x + 1)$ $\sin x(2\cos x - 1)$
(b)	$\dfrac{\cos 2x + 1}{\sin 2x} = 1$ $\dfrac{2\cos^2 x - 1 + 1}{2\sin x \cos x} = 1$ $\dfrac{2\cos^2 x}{2\sin x \cos x} = 1$ $\dfrac{\cos x}{\sin x} = 1$ $\tan x = 1$ $x = 45^\circ + k180^\circ \quad k \in \mathbb{Z}$ **Alt:** $\dfrac{\cos 2x + 1}{\sin 2x} = 1$ $2\cos^2 x - 1 + 1 = 2\sin x \cos x$ $2\cos^2 x - 2\sin x \cos x = 0$ $2\cos x(\cos x - \sin x) = 0$ $\cos x = 0$ or $\cos x - \sin x = 0$ $x \neq \pm 90^\circ + k360^\circ$ $\quad \cos x = \sin x$ $\tan x = 1$ $x = 45^\circ + k180^\circ$ $k \in \mathbb{Z}$	$2\cos^2 x - 1$ $2\sin x \cos x$ $2\cos^2$ x $\dfrac{\cos x}{\sin x}$ $\tan x = 1$ $x = 45^\circ + k180^\circ$ $2\cos^2 x - 1$ or $1 - 2\sin^2 x$ or $\cos^2 x - \sin^2 x$ $2\sin x \cos x$ $\cos x - \sin x = 0$ $x \neq \pm 90^\circ + k360^\circ$ $\tan x = 1$ $x = 45^\circ + k180^\circ$

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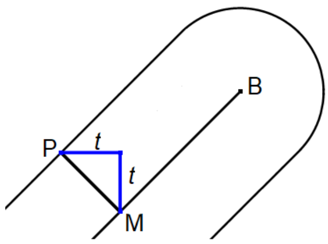
QUESTION 13

(a)	$E(9;10)$ $\therefore b = 10$ $a = 9 - 25$ $a = -16$ $\therefore C(-16;10)$	$E(9;10)$ $b = 10$ $a = -16$
(b)	$\hat{D} = 90°\ (\tan \perp \text{rad})$ $CD = 15(\text{pythag})$ $\therefore (x+16)^2 + (y-10)^2 = 225$	$\hat{D} = 90°\ (\tan \perp \text{rad})$ $CD = 15$ $(x+16)^2 + (y-10)^2 = 225$
(c)	$r \in (0;10)$ or $r \in (40;\infty)$	$r \in (0;10)$ $r \in (40;\infty)$

QUESTION 14

(a)	$M_{AB}(8;4)$ $m_{AB} = 1$ $\therefore m_{PQ} = -1$ PQ: $y - 4 = -1(x-8)$ $y = -x + 12$ $MP^2 = (x-8)^2 + (y-4)^2$ *sub* MP *and* $y = -x + 12$ $(2\sqrt{2})^2 = (x-8)^2 + (-x+12-4)^2$ $(2\sqrt{2})^2 = (x-8)^2 + (-x+8)^2$ $8 = x^2 - 16x + 64 + x^2 - 16x + 64$ $0 = 2x^2 - 32x + 120$ $x = 10$ *or* $x = 6$ $\therefore (6;6)$ and $(10;2)$ **Alternate:** $M_{AB}(8;4)$ $m_{AB} = 1$ $\therefore m_{PQ} = -1$ $\therefore \frac{\Delta y}{\Delta x} = \frac{t}{t}$  $t^2 + t^2 = (2\sqrt{2})^2$ $t = 2$ $(6;6)$ and $(10;2)$	$M_{AB}(8;4)$ $m_{AB} = 1$ and $m_{PQ} = -1$ PQ: $y = -x + 12$ $MP^2 = (x-8)^2 + (y-4)^2$ $0 = 2x^2 - 32x + 120$ $(6;6)$ $(10;2)$
(b)	$V = \pi r^2 H + \frac{4}{3}\pi r^3$ $V = \pi(2\sqrt{2})^2 \times 8\sqrt{2} + \frac{4}{3}\pi(2\sqrt{2})^3$ $V = 379{,}13$ *units*^{3}	$V = \pi r^2 H + \frac{4}{3}\pi r^3$ $H = 8\sqrt{2}$ $r = 2\sqrt{2}$ $V = 379{,}13$ *units*^{3}

Total: 150 marks