NATIONAL SENIOR CERTIFICATE EXAMINATION
MAY 2025

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) (1)

$$a = 34$$
$$d = -9$$
$$\therefore T_{16} = 34 + (15)(-9)$$
$$= -101$$

(2)

$$S_{128} = \frac{128}{2}\left[2(34) + 127(-9)\right]$$
$$= -68\ 800$$

(b)

$$\sum_{n=0}^{1} p^{3n} = 1001$$

$$p^{3(0)} + p^{3(1)} = 1001$$
$$\therefore p^0 + p^3 = 1001$$
$$\therefore p^3 = 1000$$
$$\therefore p = 10$$

(c) (1) 20 ; 27

(2)

$$2a = -2$$
$$\therefore a = -1$$
$$3(-1) + b = 13$$
$$\therefore b = 16$$
$$(-1) + (16) + c = -13$$
$$\therefore c = -28$$
$$T_n = -n^2 + 16n - 28$$

(3)

$$n = -\frac{b}{2a}$$
$$= -\frac{16}{2(-1)}$$
$$= 8$$

OR

$$\frac{dT_n}{dn} = -2n + 16 = 0$$
$$\therefore 2n = 16$$
$$\therefore n = 8$$

OR

–13 ; 0 ; 11 ; 20 ; 27 ; 32 ; 35 ; 36 ; 35 ; 32 ; …
$\therefore n = 8$

QUESTION 2

(a) $p'(x) = \lim_{h \to 0} \frac{p(x+h) - p(x)}{h}$

$$= \lim_{h \to 0} \frac{5(x+h)^2 - 5x^2}{h}$$
$$= \lim_{h \to 0} \frac{5x^2 + 10xh + 5h^2 - 5x^2}{h}$$
$$= \lim_{h \to 0} \frac{h(10x + 5h)}{h}$$
$$= \lim_{h \to 0} (10x + 5h)$$
$$= 10x$$

(b) (1) $y = \sqrt{x} + \frac{1}{x^3} + 1$

$$= x^{\frac{1}{2}} + x^{-3} + 1$$
$$\therefore \frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}} - 3x^{-4}$$

(2) $D_x \left(\frac{x^2 - 25}{x - 5} \right)$

$$= D_x \left(\frac{(x-5)(x+5)}{x-5} \right)$$
$$= D_x (x+5)$$
$$= 1$$

(c) $k(x) = x^3 + 6x^2 - 7$

$$\therefore k'(x) = 3x^2 + 12x$$
$$\therefore k''(x) = 6x + 12$$
$$\therefore 6x + 12 > 0$$
$$\therefore 6x > -12$$
$$\therefore x > -2$$

QUESTION 3

(a)
$$b = \frac{8}{-8}$$
$$\therefore b = -1$$

(b) $x \neq 0 \quad x \in \mathbb{R}$

(c) $y = x$
$y = -x$

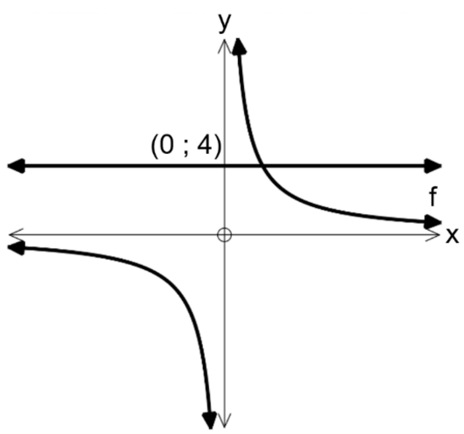
(d) $x > 0$

(e) (1) $y = -\frac{8}{x}$

(2) $y = \frac{8}{x} - 10$

(3)
$$x = \frac{8}{y}$$
$$\therefore xy = 8$$
$$\therefore y = \frac{8}{x}$$

(f) (1) y int
horizontal line



(2) $f(x) = g(x)$
$$\frac{8}{x} = 4$$
$$\therefore 8 = 4x$$
$$\therefore x = 2$$
$$\therefore (2\ ;\ 4)$$

QUESTION 4

(a)

$$120\ 000=\frac{x\left[1-\left(1+\frac{0{,}0957}{12}\right)^{-60}\right]}{\frac{0{,}0957}{12}}$$

$$\therefore \frac{120\ 000\left(\frac{0{,}0957}{12}\right)}{\left[1-\left(1+\frac{0{,}0957}{12}\right)^{-60}\right]}=x$$

$$\therefore x=\text{R}2\ 524{,}33$$

(b)

$$1+i_{eff}=\left(1+\frac{i^m}{m}\right)^m$$

$$\therefore 1+i_{eff}=\left(1+\frac{0{,}0957}{12}\right)^{12}$$

$$\therefore i_{eff}=0{,}1$$

$$=10\%$$

QUESTION 5

(a) $y > -9{,}5 \quad y \in \mathbb{R}$

(b)

$$b=-9{,}5$$

$$-8=a^{-1}-9{,}5$$

$$\therefore \frac{3}{2}=a^{-1}$$

$$a=\frac{2}{3}$$

(c)

$$0=\left(\frac{2}{3}\right)^x-9{,}5$$

$$\therefore 9{,}5=\left(\frac{2}{3}\right)^x$$

$$\therefore x=\log_{\frac{2}{3}} 9{,}5$$

$$\therefore x=-5{,}55$$

$$\therefore (-5{,}55\ ;\ 0)$$

QUESTION 6

(a) (1) $P(S \cup T) = 0,7 + 0,2$
$= 0,9$

(2) $P(S \cap T) = 0,7 \times 0,2$
$= 0,14$
$P(S \cup T) = P(S) + P(T) - P(S \cap T)$
$= 0,7 + 0,2 - 0,14$
$= 0,76$

(b) (1) $= \dfrac{7!}{2!}$
$= 2\ 520$

(2) $= 5! \div 2\ 520$
$= 120 \div 2\ 520$
$= \dfrac{1}{21}$
$\approx 0,05$

OR

$= \dfrac{2}{7} \times \dfrac{1}{6}$
$= \dfrac{1}{21}$
$\approx 0,05$

(3) $= 6! \div 2\ 520$
$= \dfrac{2}{7}$
$\approx 0,29$

SECTION B

QUESTION 7

(a) (1) $11 - 2x + \sqrt{2x^2 - 29x + 114} = 0$

$\sqrt{2x^2 - 29x + 114} = 2x - 11$
$\therefore 2x^2 - 29x + 114 = (2x - 11)^2$
$\therefore 2x^2 - 29x + 114 = 4x^2 - 44x + 121$
$\therefore 0 = 2x^2 - 15x + 7$
$\therefore 0 = (x - 7)(2x - 1)$
$\therefore x = 7 \quad \text{or} \quad x \neq \frac{1}{2}$

(2) $0 = 5^{2x} - 9(5^x) - 10$
$\therefore 0 = (5^x - 10)(5^x + 1)$
$\therefore 5^x = 10 \quad \text{or} \quad 5^x \neq -1$
$\therefore x = \log_5 10$
$\therefore x = 1{,}43$

OR

let $k = 5^x$
$\therefore 0 = k^2 - 9k - 10$
$\therefore 0 = (k - 10)(k + 1)$
$\therefore k = 10 \quad \text{or} \quad k \neq -1$
$\therefore 5^x = 10 \quad \text{or} \quad 5^x \neq -1$
$\therefore x = \log_5 10$
$\therefore x = 1{,}43$

(3) $x^{\frac{8}{3}} - 9x^{\frac{10}{3}} = 0$
$\therefore x^{\frac{8}{3}}\left(1 - 9x^{\frac{2}{3}}\right) = 0$
$\therefore x^{\frac{8}{3}} = 0 \quad \text{or} \quad 1 = 9x^{\frac{2}{3}}$
$\therefore x = 0 \quad \text{or} \quad \frac{1}{9} = x^{\frac{2}{3}}$
$\therefore \pm\frac{1}{3} = x^{\frac{1}{3}}$
$\therefore \pm\frac{1}{27} = x$

(b)

$$k = \frac{x}{2}(x-4)$$

$$\therefore 0 = \frac{1}{2}x^2 - 2x - k$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4\left(\frac{1}{2}\right)(-k)}}{2\left(\frac{1}{2}\right)}$$

$$= 2 \pm \sqrt{4 + 2k}$$

$$\therefore 4 + 2k \geq 0$$

$$\therefore 2k \geq -4$$

$$\therefore k \geq -2$$

OR

$$0 = x^2 - 4x - 2k$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-2k)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 8k}}{2}$$

$$\therefore 16 + 8k \geq 0$$

$$\therefore 8k \geq -16$$

$$\therefore k \geq -2$$

QUESTION 8

(a)

$$x\left(1 + \frac{0,11}{4}\right)^{64} + \frac{x}{2}\left(1 + \frac{0,11}{4}\right)^{32} = 200\ 000$$

$$\therefore x\left[\left(1 + \frac{0,11}{4}\right)^{64} + \frac{1}{2}\left(1 + \frac{0,11}{4}\right)^{32}\right] = 200\ 000$$

$$\therefore x = \frac{200\ 000}{\left(1 + \frac{0,11}{4}\right)^{64} + \frac{1}{2}\left(1 + \frac{0,11}{4}\right)^{32}}$$

$$\therefore x = \text{R}29\ 124{,}20$$

(b)

$$F_v = \frac{800\left[\left(1 + \frac{0,09}{12}\right)^{121} - 1\right]}{\frac{0,09}{12}}\left(1 + \frac{0,09}{12}\right)^{72}$$

$$= \text{R}268\ 481{,}18$$

QUESTION 9

(a) $h + r = 30$

$\therefore h = 30 - r$

$$V = \frac{1}{3}\pi r^2(30 - r)$$

$$= 10\pi r^2 - \frac{\pi}{3}r^3$$

(b) $\frac{dV}{dr} = 20\pi r - \pi r^2$

$\therefore 0 = \pi r(20 - r)$

$\therefore r \neq 0 \quad \text{or} \quad r = 20$

$$V = 10\pi(20)^2 - \frac{\pi}{3}(20)^3$$

$$= \frac{4\ 000\pi}{3}\ \text{cm}^3$$

$$\approx 4\ 188{,}79\ \text{cm}^3$$

(c) (1) $(20\ ;\ 4188{,}79)$

(2)

$$0 = 10\pi x^2 - \frac{\pi}{3}x^3$$

$$\therefore 0 = \frac{1}{3}\pi x^2(30 - x)$$

$\therefore x = 0 \quad \text{or} \quad x = 30$

$$f'(x) = 20\pi(30) - \pi(30)^2$$

$$= -300\pi$$

$$\approx -942{,}48$$

subs: $0 = -300\pi(30) + c$

$\therefore c = 9\ 000\pi$

$\therefore y = -300\pi x + 9\ 000\pi$

$\therefore a = -300 \quad \text{and} \quad b = 9\ 000$

QUESTION 10

(a) $x = 0$
$x = 5$

(b) $5 < x \leq 6$

(c) $\log_2 x - \log_2(x-5) = 1$

$$\therefore \log_2\left(\frac{x}{x-5}\right) = 1$$

$$\therefore 2 = \frac{x}{x-5}$$

$$\therefore 2x - 10 = x$$

$$\therefore x = 10$$

QUESTION 11

(a) $0 = -(y+1)(y-4)(y-7)$

$\therefore y = -1$ or $y = 4$ or $y = 7$

$\therefore$ AC = 8 units

(b) $y = -(x+1)(x-4)(x-7)$

(c) $y = -(x+1)(x-4)(x-7)$

$\therefore y = (-x-1)(x^2 - 11x + 28)$

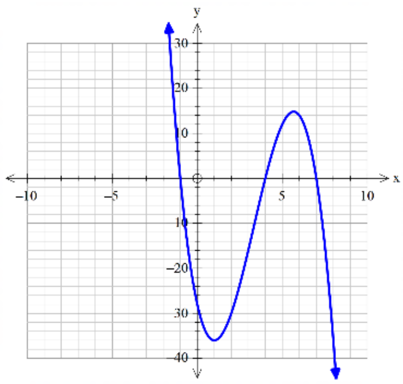
$\therefore y = -x^3 + 10x^2 - 17x - 28$

$$\frac{dy}{dx} = -3x^2 + 20x - 17 = 0$$

$\therefore 3x^2 - 20x + 17 = 0$

$\therefore (3x-17)(x-1) = 0$

$$\therefore x = \frac{17}{3} \quad \text{or} \quad x = 1$$



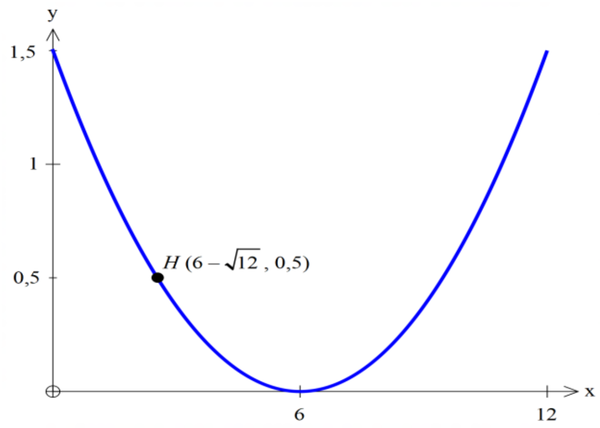
subs $y = -(1+1)(1-4)(1-7)$

$\therefore y = -36$

$\therefore$ MN is 36 units

QUESTION 12

(a) TP at (6 ; 0)
y intercept at (0 ; 1,5)
$\therefore y = a(x-6)^2 + 0$



subs (0 ; 1,5): $1{,}5 = a(0-6)^2$

$\therefore \frac{1{,}5}{36} = a$

$\therefore a = \frac{1}{24}$

$\therefore y = \frac{1}{24}(x-6)^2$

subs y = 0,5: $0{,}5 = \frac{1}{24}(x-6)^2$

$\therefore 12 = (x-6)^2$

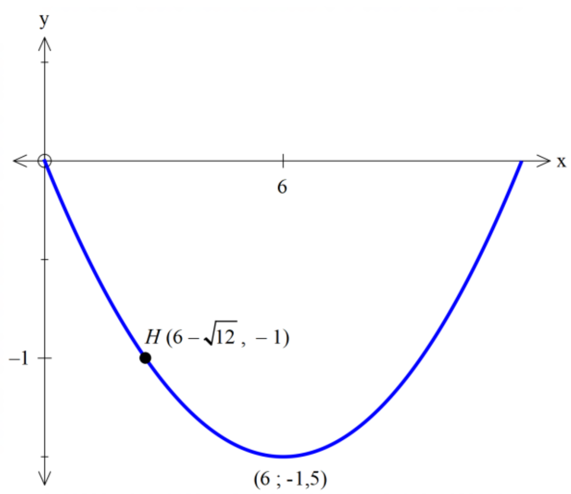
$\therefore \pm 2\sqrt{3} = x - 6$

$\therefore x = 6 - 2\sqrt{3}$ or $x = 6 + 2\sqrt{3}$

So the horizontal distance from H to B is $2\sqrt{3}$ metres

OR

TP at (6 ; –1,5)
y intercept at (0 ; 0)
$\therefore y = a(x-6)^2 - 1{,}5$



subs (0 ; 0): $0 = a(0-6)^2 - 1{,}5$

$\therefore 1{,}5 = 36a$

$\therefore a = \frac{1}{24}$

$\therefore y = \frac{1}{24}(x-6)^2 - 1{,}5$

subs y = –1: $-1 = \frac{1}{24}(x-6)^2 - 1{,}5$

$\therefore \frac{1}{2} = \frac{1}{24}(x-6)^2$

$\therefore 12 = (x-6)^2$

$\therefore \pm 2\sqrt{3} = x - 6$

$\therefore x = 6 - 2\sqrt{3}$ or $x = 6 + 2\sqrt{3}$

So the horizontal distance from H to B is $2\sqrt{3}$ metres

OR

TP and y intercept at (0 ; –1,5)
$\therefore y = ax^2 - 1,5$
subs (6 ; 0): $y = a\left(6^2\right) - 1,5$
$\therefore 1,5 = 36a$
$\therefore a = \frac{1}{24}$

$\therefore y = \frac{1}{24}x^2 - 1,5$

subs y = –1: $-1 = \frac{1}{24}x^2 - 1,5$
$\therefore \frac{1}{2} = \frac{1}{24}x^2$
$\therefore 12 = x^2$
$\therefore x = \pm 2\sqrt{3}$

So the horizontal distance from H to B is $2\sqrt{3}$ metres

(b)

$y = \frac{1}{24}(x^2 - 12x + 36)$

$\therefore y = \frac{1}{24}x^2 - \frac{1}{2}x + \frac{3}{2}$

$\frac{dy}{dx} = \frac{1}{12}x - \frac{1}{2}$

subs x = 0: $\frac{dy}{dx} = \frac{1}{12}(0) - \frac{1}{2} = -\frac{1}{2}$ **OR** subs x = 12: $\frac{dy}{dx} = \frac{1}{12}(12) - \frac{1}{2} = \frac{1}{2}$

OR

$y = \frac{1}{24}(x^2 - 12x + 36) - 1,5$

$\therefore y = \frac{1}{24}x^2 - \frac{1}{2}x$

$\frac{dy}{dx} = \frac{1}{12}x - \frac{1}{2}$

subs x = 0: $\frac{dy}{dx} = \frac{1}{12}(0) - \frac{1}{2} = -\frac{1}{2}$ **OR** subs x = 12: $\frac{dy}{dx} = \frac{1}{12}(12) - \frac{1}{2} = \frac{1}{2}$

$y = \frac{1}{24}x^2 - 1,5$ hence $\frac{dy}{dx} = \frac{x}{12}$ $\therefore m = \frac{6}{12} = \frac{1}{2}$

QUESTION 13

(a)

$$AC = 1 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$$
$$= 1 + S_\infty$$
$$= 1 + \frac{1}{1-\frac{1}{2}}$$
$$= 1 + 2$$
$$= 3 \text{ metres}$$

(b)

$$BC^2 = 3^2 - 1^2 \qquad \text{(Pythag)}$$
$$\therefore BC = 2\sqrt{2} \text{ metres}$$
$$\cos A = \frac{1}{3}$$
$$\therefore A = 70{,}53°$$

$$\text{Area} = \frac{1}{2}\left(2\sqrt{2}\right)(1) - \frac{70{,}53°}{360°}\left[\pi(1)^2\right] - \left[\frac{1}{2}\pi\left(\frac{1}{2}\right)^2 + \frac{1}{2}\pi\left(\frac{1}{4}\right)^2 + \frac{1}{2}\pi\left(\frac{1}{8}\right)^2 + \ldots\right]$$
$$= \sqrt{2} - \frac{70{,}53°\pi}{360°} - \frac{1}{2}\left[\frac{\pi}{4} + \frac{\pi}{16} + \frac{\pi}{64} + \ldots\right]$$
$$= \sqrt{2} - \frac{70{,}53°\pi}{360°} - \frac{1}{2}\left[\frac{\frac{\pi}{4}}{1-\frac{1}{4}}\right]$$
$$= \sqrt{2} - \frac{70{,}53°\pi}{360°} - \frac{\pi}{6}$$
$$= 0{,}28 \text{ m}^2$$

Total: 150 marks