EXAMINATION NUMBER

Time: 3 hours   150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1.   This question paper consists of 31 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.

2.   Read the questions carefully.

3.   **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided on the question paper.**

4.   Diagrams are not necessarily drawn to scale.

5.   You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.

6.   Ensure that your calculator is in **DEGREE** mode.

7.   Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**

8.   Round off to **TWO DECIMAL PLACES** unless otherwise stated.

9.   It is in your own interest to write legibly and to present your work neatly.

10.   One blank page (page 30) and extra graphs (page 31) are included at the end of the paper. If you run out of space for a question, use this page. Clearly indicate the number of your answer should you use this extra space.
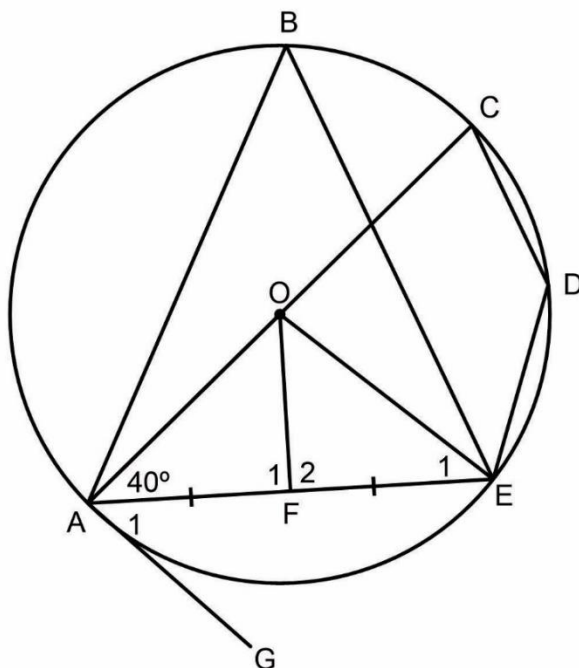

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	TOTAL
12	13	13	8	15	9	11	9	14	5	8	14	8	11	150

SECTION A

QUESTION 1   (7)

(a)   In the diagram:

- O is the centre of the circle ABCDE.
- AF = FE.
- AG is a tangent to the circle at A.
- $O\hat{A}E = 40°$.

Complete the table by filling in the appropriate reason for the given statements, in the order given below.

Statement	Reason
OA = OE	
$\hat{E}_1 = 40°$	
$\hat{D} = 140°$	
$\hat{F}_1 = 90°$	
$\hat{A}_1 = 50°$	
$\hat{B} = 50°$	
$A\hat{O}E = 100°$	

(7)

(b) Use the diagram below to prove the statement that says:

'The acute angle formed by a chord and a tangent at the point of contact is equal to the angle in the alternate segment.'
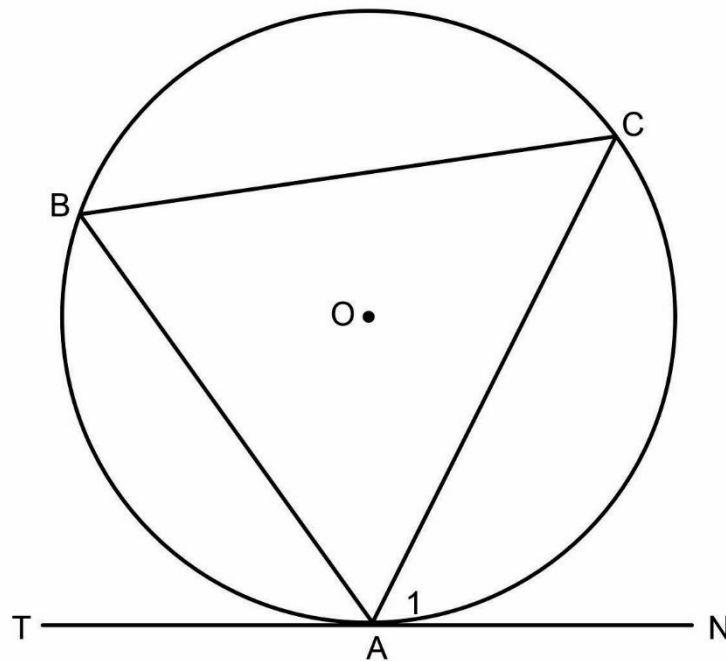
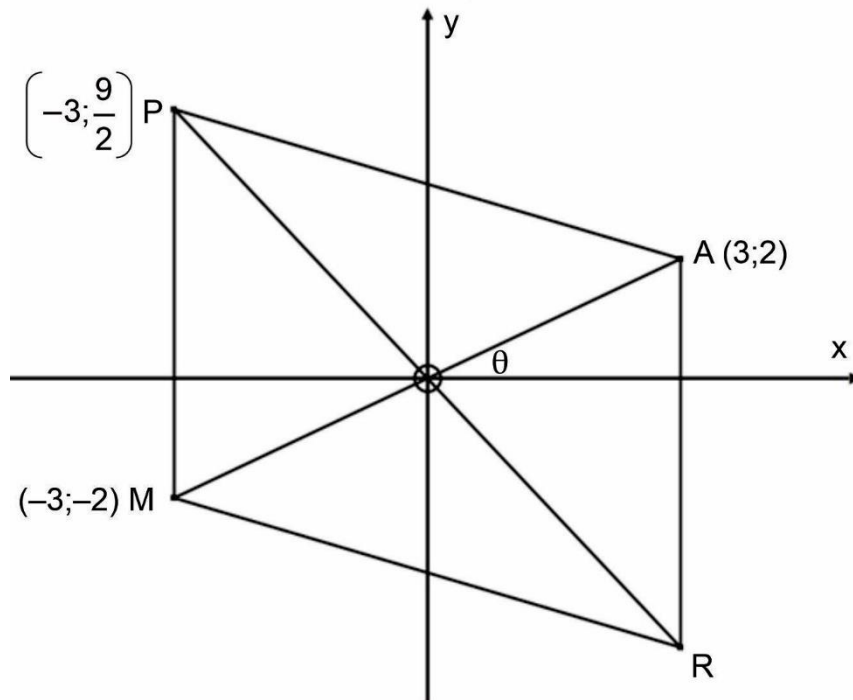Required to prove: $\hat{A}_1 = \hat{B}$

Construction: _____

Proof:

(5)
[12]

QUESTION 2

In the diagram:

- PARM is a parallelogram with $P\left(-3;\dfrac{9}{2}\right)$, $A(3;2)$ and $M(-3;-2)$.

- The diagonals of the parallelogram intersect at the origin.
- θ is the angle of inclination of line AM.

(a)   Determine the co-ordinates of R.

                                                                                                (2)

(b)   Determine, in simplest surd form, the distance AM.

                                                                                                (2)

(c)   Determine the size of θ.

(3)

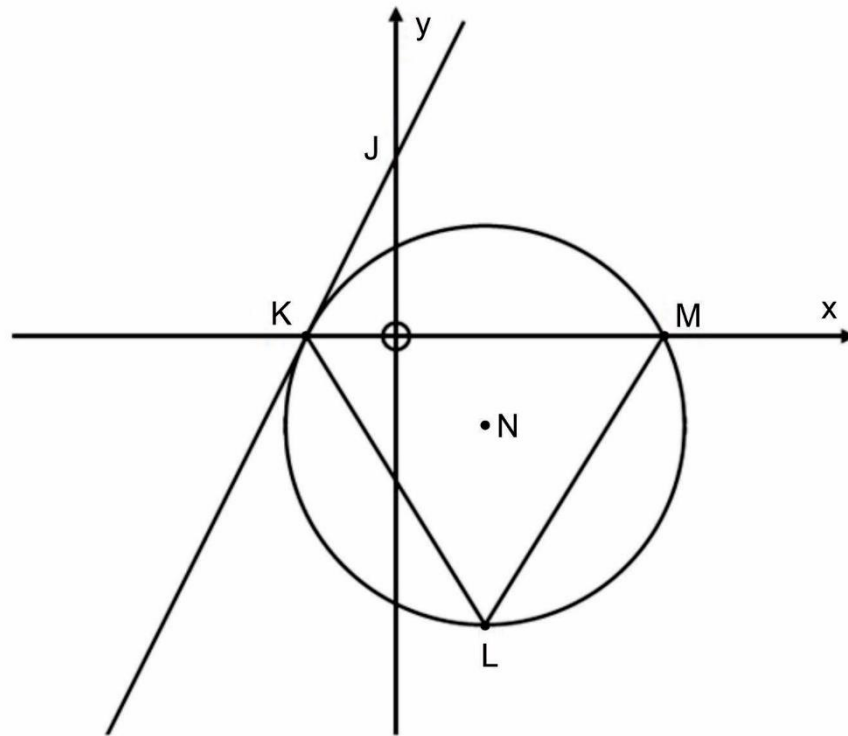(d)   Determine the equation of the line parallel to AM passing through P, in the form $y = mx + c$.

(3)

(e)   Prove that PARM is a rhombus, an appropriate reason must be given in the conclusion.

(3)
[13]

QUESTION 3

Given:

- Circle with centre N.
- The equation of the circle is $x^2 - 2x + y^2 + 2y - 3 = 0$.
- JK is a tangent at K.

(a)   Determine the centre and the radius length of circle N.

(3)

(b)   Determine the co-ordinate K.

(4)

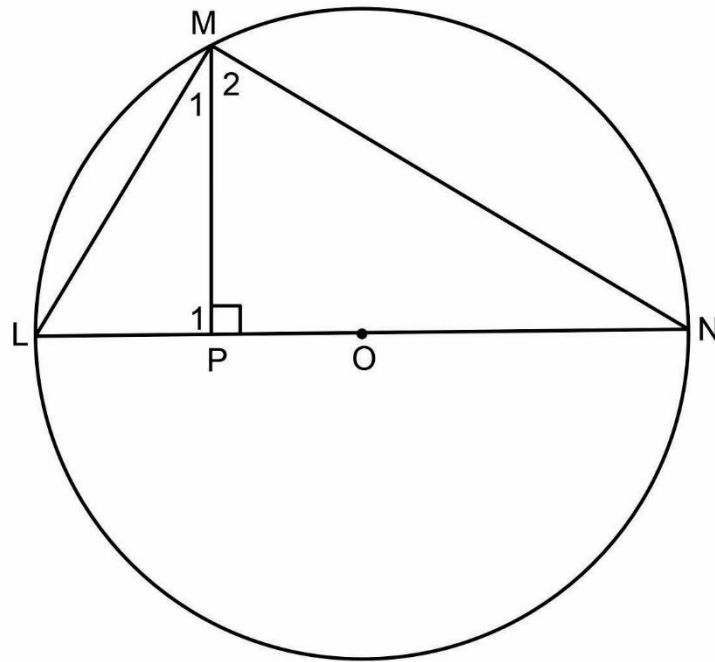(c)   Determine the equation of the tangent JK.

(3)

(d)   Determine the size of $K\hat{L}M$, a relevant reason must be given.

(3)
[13]

QUESTION 4

Given circle centre O with:

- Diameter of 8 units.
- LM = 4 units.
- $LN \perp MP$.

State all relevant reasons with your statements.

(a)   Determine: $\hat{M}$

(2)

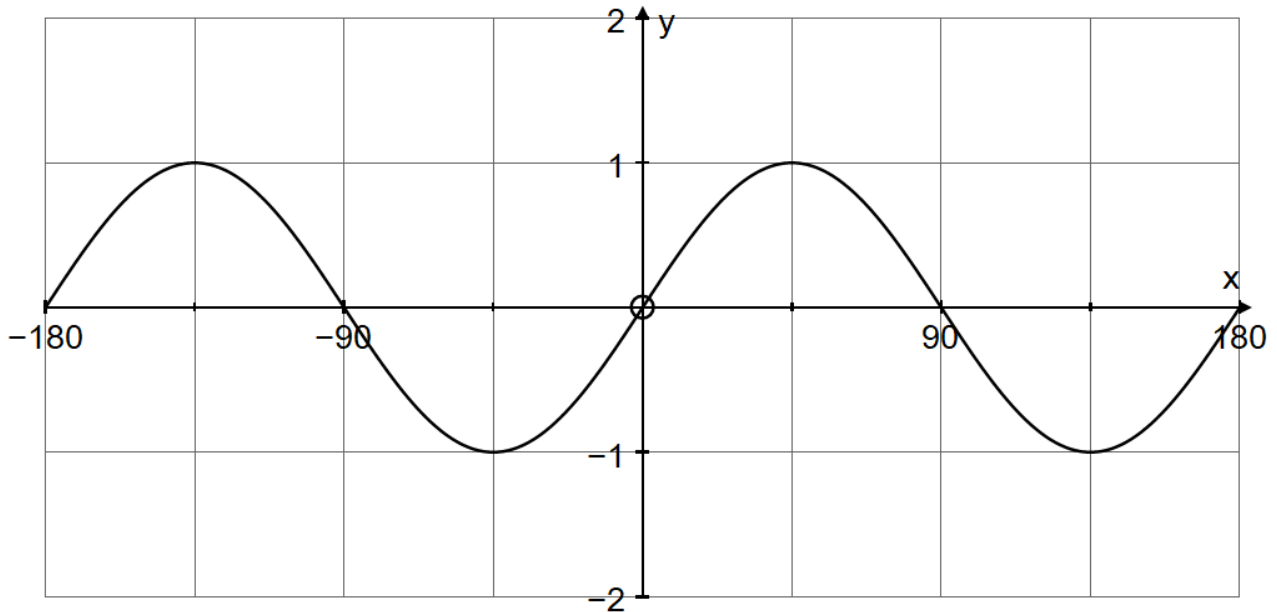(b)   Prove: $\triangle LMN \, ||| \, \triangle LPM$

(3)

(c)   Determine: LP

(3)
[8]

QUESTION 5

Sketched is the graph of $f(x) = \sin bx$ for $x \in [-180°; 180°]$.

(a)   Determine the value of b.

(1)

(b)   State the period of f.

(1)

(c)   Sketch the graph of $g(x) = -2\cos x$ on the **same set** of axes as f.

(3)

(d)   State the amplitude of g.

(1)

(e)   State the general solution for $f(x) = g(x)$.

(2)

(f)   If $x \in [-180°; 180°]$, determine the values for x where:

   (1) $f(x) \geq g(x)$

(1)

   (2) $\dfrac{f(x)}{g(x)} < 0$

(2)

   (3) $g'(x) > 0$

(2)

(g)   If g is shifted 2 units up and 45⁰ to the right, write down the new equation for g in the form $y = ...$

(2)
[15]

QUESTION 6

(a)   Given: $\sin 40° = \dfrac{p}{4}$

Determine the values of the following in terms of p:

(1) $\sin 50°.\cos 10° - \cos 50°.\sin 10°$

(2)

(2) $\cos^2 20° - \sin^2 20°$

(3)

(3) $\tan 50°$

(1)

(b)   In $\triangle$ABC :

- AB = 10 units.
- BC = 12 units.
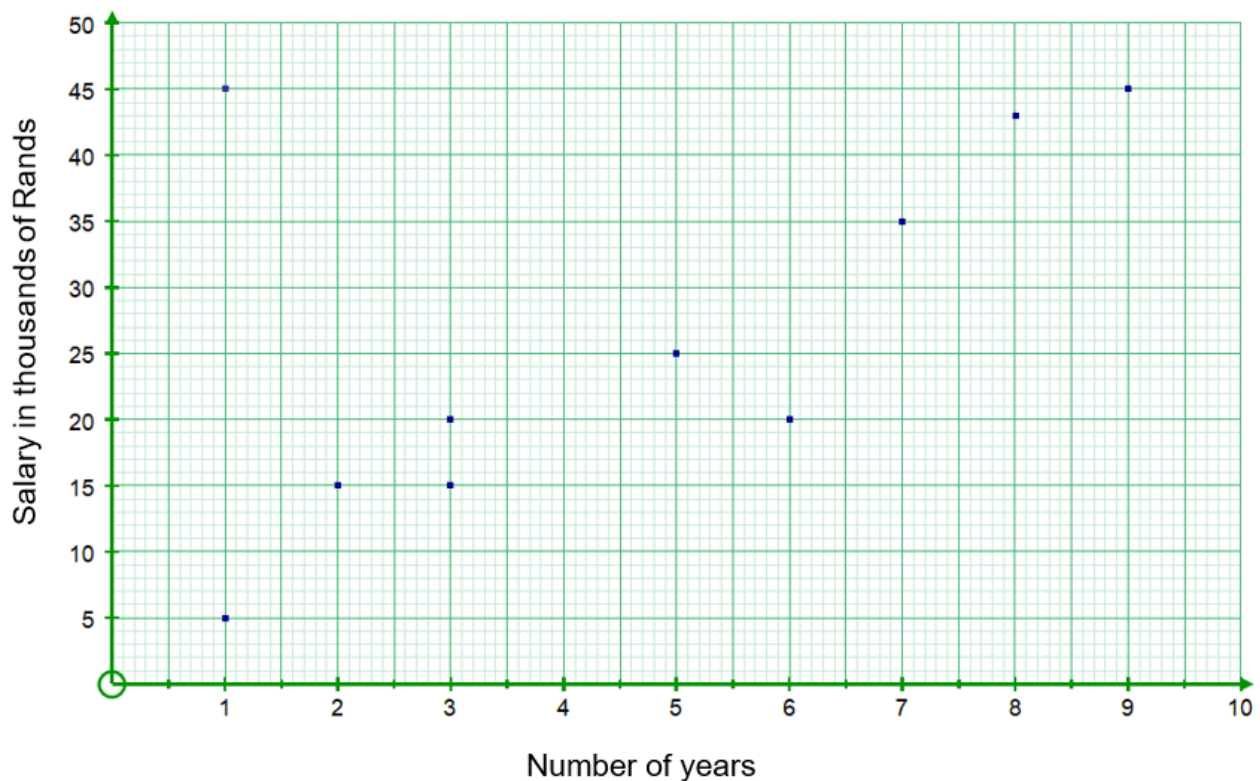- A$\hat{\text{B}}$C = 60°.

Determine the length of AC.

(3)
[9]

QUESTION 7

The table and scatter plot below shows the salaries of 10 employees at a certain company. Where x is the number of years the employee has worked at the company and y is the salary of the employee per month, in thousands of rands.

x	1	1	2	3	3	5	6	7	8	9
y	45	5	15	20	15	25	20	35	43	45

(a) Use your calculator to determine the equation of the line of best fit in the form $y = A + Bx$.

(2)

(b) Draw the line of best fit on the scatter plot above, showing the y intercept and the co-ordinate $(\overline{x}\,;\overline{y})$.

(4)

(c)   By using your answer in (b) determine the salary of an employee who has been working at the company for 4 years.

(2)

(d)   Calculate the correlation coefficient for the above data.

(1)

(e)   Circle the outlier **on the graph**.   (1)

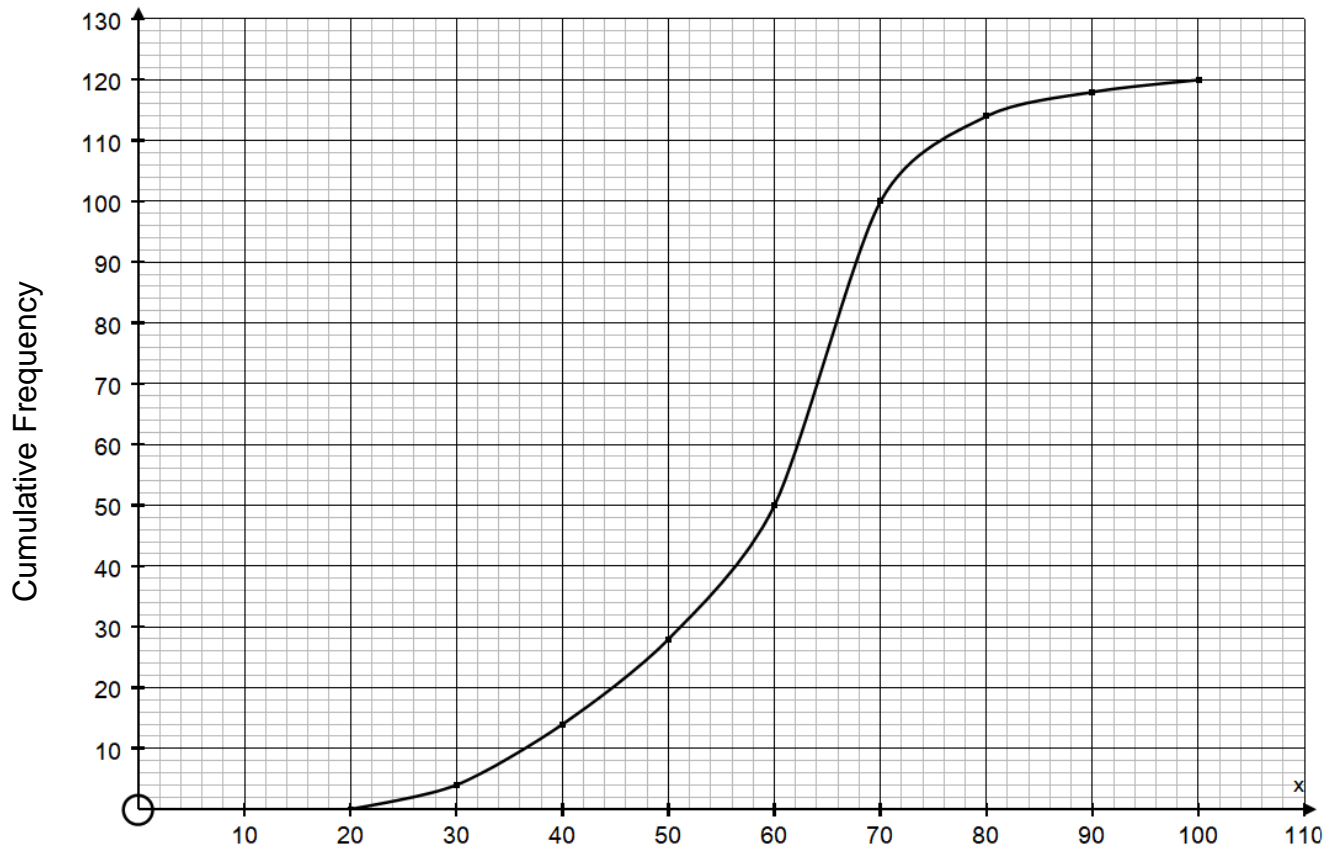(f)   If this outlier is removed from the data, discuss the effect it will have on the correlation coefficient.

(1)
[11]

81 marks

SECTION B

QUESTION 8

(a)   Study the cumulative frequency curve below.

If the data for the class intervals above 70 are removed from the curve, determine the difference between the original median and the new median result.

(3)

(b)   Study the box-and-whisker plot.

Will the mean be bigger or smaller than the median? Explain.

(2)

(c)   Given three numbers 4, 8 and *p*.

Consider the table below and hence determine the numerical value of *p* and the standard deviation of the three numbers.
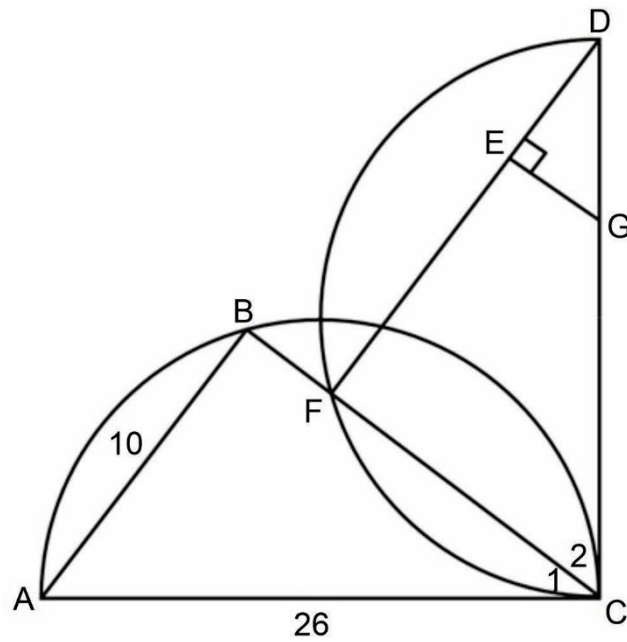
x	$x - \bar{x}$	$(x - \bar{x})^2$
4	−3	
8		
p		

(4)
[9]

QUESTION 9

Given:

- Two equal semi circles.
- AC = CD = 26 units.
- AB = 10 units.
- AC and DC are tangents to circles CFD and ABC respectively.
- GE $\perp$ FD.

State all relevant reasons with your statements.

(a)   Prove that EG $\|$ FC.

(3)

(b)   Prove that ∆ABC ≡ ∆CFD and hence that CF is 10 units.

(6)

(c)   If it is further given that 3BA = 5DE, show that FE = 18 units.

(2)

(d)   Determine the value of GC.

(3)
[14]

QUESTION 10

In the diagram:

- A, B, C, H and D all lie on the circumference of the circle.
- BGEHF is a straight line.
- $\dfrac{AC}{DF} = \dfrac{CD}{FE} = \dfrac{AD}{DE}$

State all relevant reasons with your statements.

(a)   Give a reason why $\triangle ACD \;|||\; \triangle DFE$.

(1)

(b)   Prove that DF is a tangent to the circle at D.

(2)

(c)   Prove that GCFD is a cyclic quadrilateral.

(2)
[5]

QUESTION 11

In the diagram:

- $\triangle ABC$ is in the horizontal plane, with BC = 10 units.
- $A\hat{C}B = 2\theta - 90°$ and $A\hat{B}C = 180° - \theta$.
- The angle of inclination from A to D is θ.
- $\triangle ACD$ is in the vertical plane, with CD = 15 units.
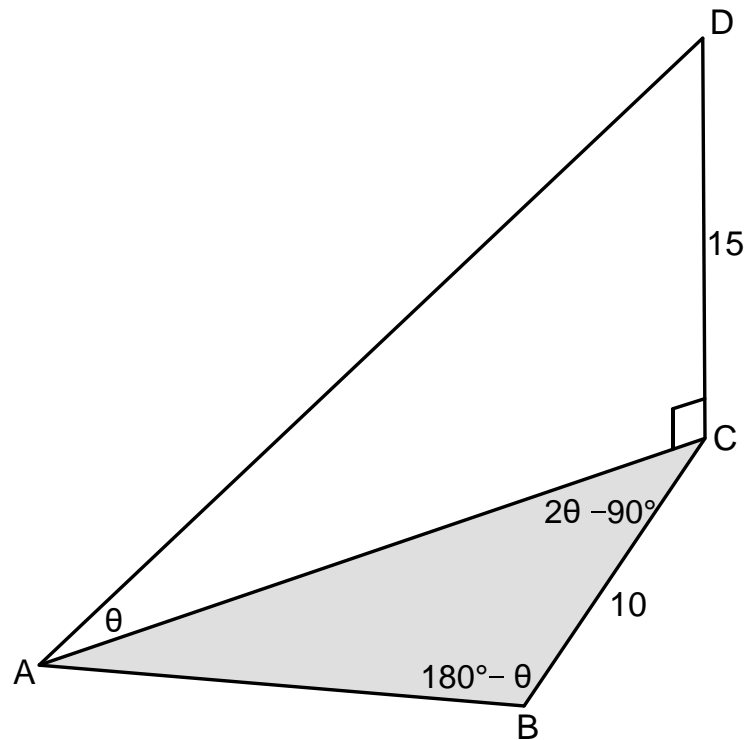
(a)   Show that $AC = 10\tan\theta$.

(4)

(b)   Hence, determine the size of θ if $\theta \in [0°;90°]$.

(4)
[8]

QUESTION 12

(a) Prove the following using the fundamental trigonometric identities:

$$\frac{\cos(360° - 2x) - \cos(180° + x)}{\sin 2x - \cos(90° - x)} = \frac{\cos x + 1}{\sin x}$$

(7)

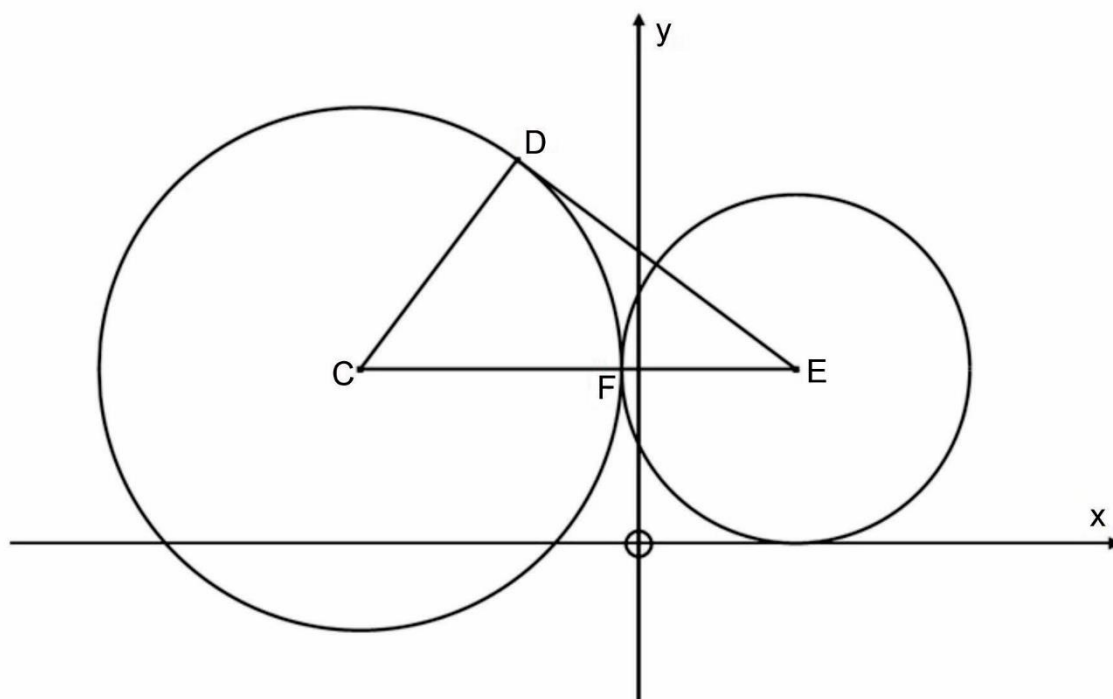(b)   Determine the general solution for:

$$\frac{\cos 2x + 1}{\sin 2x} = 1$$

(7)
[14]

QUESTION 13

Given:

- Circle with centre $C(a;b)$.
- Circle with centre E has the equation $(x-9)^2 + (y-10)^2 = r^2$.
- CE = 25 units and is parallel to the x axis.
- F is the point of tangency between the two circles.

(a)   Determine the co-ordinates of C. (3)

(b)   If it is further given that ED = 20 units and is a tangent to the larger circle at D, determine the equation of the circle with centre C.

(3)

(c)   Determine the possible values of the radius (r) of circle E, such that the two circles will not touch or intersect.
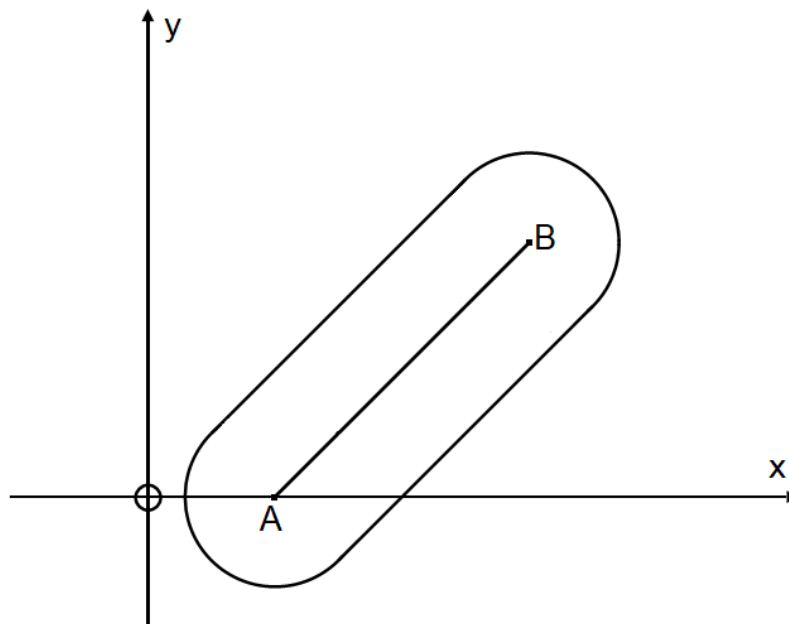
(2)
[8]

QUESTION 14

A stadium is represented on the Cartesian plane.

> A stadium is a two-dimensional geometric shape constructed of a rectangle with semicircles at a pair of opposite sides.

[Source: <https://en.wikipedia.org/wiki/Stadium_(geometry)>]

All points on the circumference of the stadium are $2\sqrt{2}$ units from the line AB.
$A(4;0)$ and $B(12;8)$.

(a)   Determine the co-ordinates of P and Q, which lie on the circumference of the figure and are equidistant from A and B.

(7)

(b)   Study the definition of a capsule.

> A capsule, or stadium of revolution, is a basic three-dimensional geometric shape consisting of a cylinder with hemispherical ends.

A capsule is created when the stadium in question (a) is revolved around the line AB. With assistance of the formula $V = \frac{4}{3}\pi r^3$, determine the volume in units cubed of this capsule.
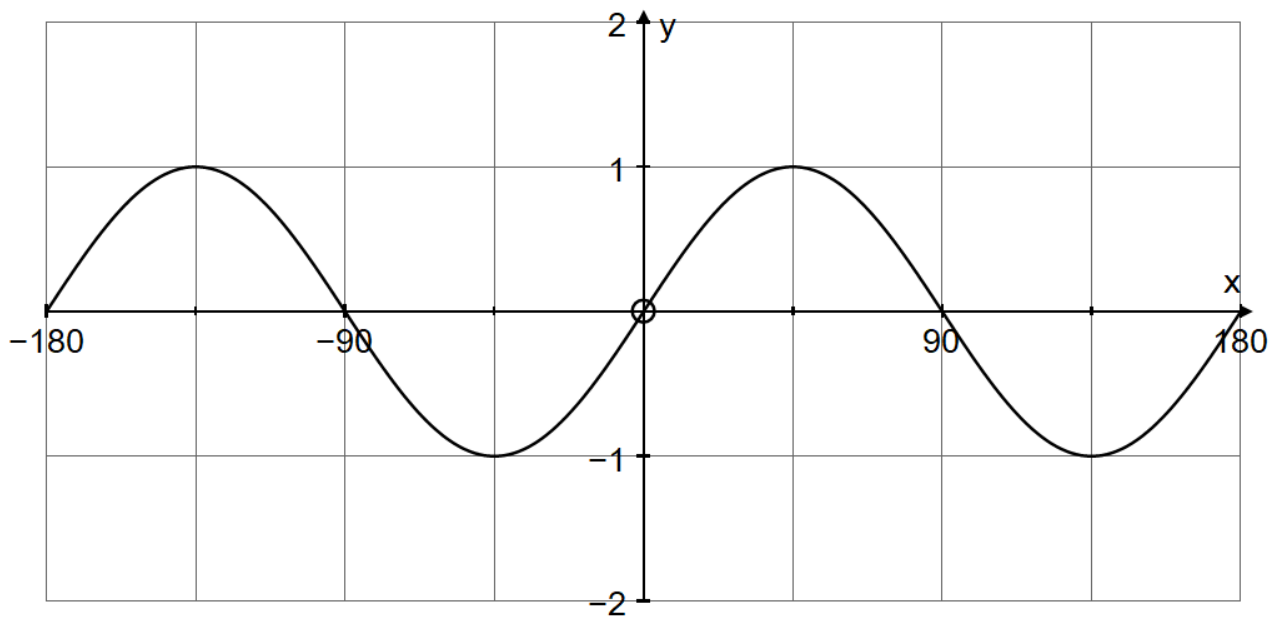
<div align="right">

(4)
[11]

</div>

<div align="right">

69 marks

</div>

<div align="right">

Total: 150 marks

</div>

ADDITIONAL SPACE (ALL QUESTIONS)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION, THAT YOU USED THE ADDITIONAL SPACE, TO ENSURE THAT ALL ANSWERS ARE MARKED.

EXTRA GRAPHS

QUESTION 5 (c)

QUESTION 7